Topology Shared Between Classical and Quantum Materials

Po-Wei Lo’s B-Exam

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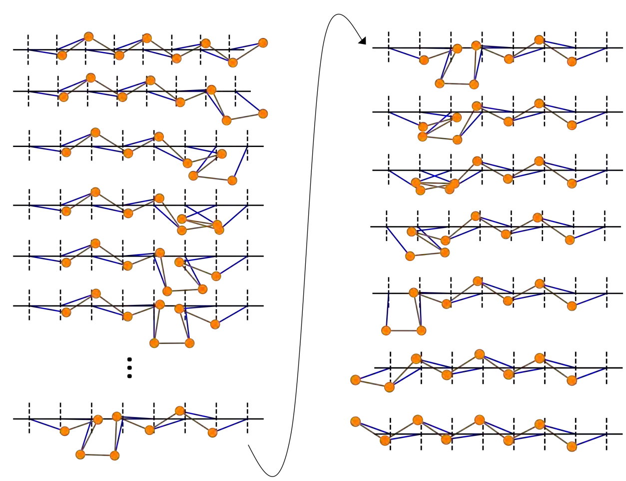
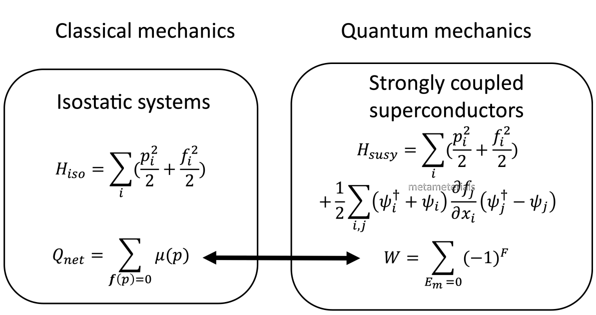
Zoom link: https://cornell.zoom.us/j/96990371612?pwd=MjRBZTljMzRvSHY3dy9IcmcvSytSUT09

The concept of topology has been widely used to classify materials. Majority of works are focused on quantum systems. Until recently, many advancements have also been made in the field of topological mechanics. However, the connections between them are still limited to the linear level of mechanical systems which are naturally nonlinear. In this work, we study the topology of nonlinear classical systems and possible connections to quantum systems.

First, we present a generic prescription of defining topological indices which accommodates nonlinear effects in mechanical systems without taking any approximation. Invoking the tools of differential geometry, a Z-valued quantity in terms of a topological index μ in differential geometry known as the Poincaré-Hopf index, that features the topological invariant of nonlinear zero modes (ZMs), is predicted. We further identify one type of topologically protected solitons that are robust to disorders. Our prescription constitutes a new direction of searching for novel topologically protected nonlinear ZMs in the future.





Secondly, we connect this topological index to the Witten index in supersymmetric quantum systems. To establish the connection, we study two topological number in isostatic mechanical systems and supersymmetric quantum systems describing a superconductor strongly coupled to anharmonic phonons, respectively. On one hand, we define Q_{net} for an isostatic mechanical system that counts the minimum number of zero-energy configurations.

On the other hand, we write a supersymmetric Hamiltonian that has a well-defined Witten index W that tells us the minimum number of zero-energy states. Finally, we show that Q_{net} = W under very general conditions. Our result suggests a direct connection between nonlinear mechanical systems and strongly coupled superconductors, and therefore points out an alternative way to understand the topology of strongly coupling quantum systems.